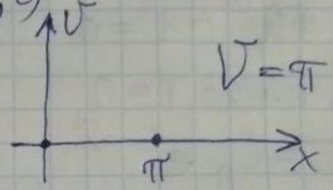
№98

$$\begin{cases} U_t = U_{xx} + U + 2\cos 3x \\ U_x(0,t) = U_x(\pi,t) = 0 \\ U(x,0) = \cos x \end{cases}$$

$U(x,t) = \upsilon(x,t) + \omega(x,t)$

$$\begin{cases} \upsilon(0,t) = 0 \\ \upsilon(\pi,t) = 0 \end{cases}$$



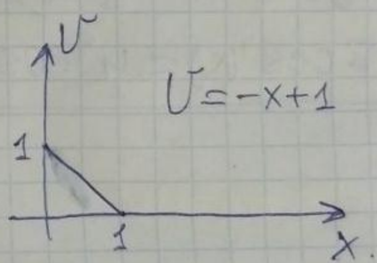
$U(x,t) = \pi + \omega(x,t)$

$\omega_t = \omega_{xx} + \pi + \omega + 2\cos 3x$

№113

$$\begin{cases} U_t = U_{xx} + U - x + 1 - U\cos^3 \frac{5\pi x}{2} \\ U_x = (0,t) = 1 \\ U(1,t) = 0 \\ U(x,0) = x - 1 \end{cases}$$

$U(x,t) = \upsilon(x,t) + \omega(x,t)$



$$\begin{cases} \upsilon(0,t) = 1 \\ \upsilon(1,t) = 0 \end{cases}$$

$U(x,t) = \upsilon(x,t) + \omega(x,t) = 1 - x + \omega(x,t)$

$\omega_t = \omega_{xx} + 1 - x + \omega - x + 1 + (1 - x + \omega)\cos^3 \frac{5\pi x}{2}$

98 $\quad u(x,t) = \frac{1}{4}\left(1 - e^{-8t}\right)\cos 3x$

113 $\quad u(x,t) = x - 1 + \sum_{n=0}^{\infty} \frac{F_n}{\pi^2 n^2 - 1}\left(1 - e^{(1-\pi^2 n^2)t}\right)\cos \pi n x,$

где

$$F_n = \frac{12000\,(-1)^n}{\pi(4n^2 - 225)(4n^2 - 25)}$$

№141

$$U_{tt} = a^2 U_{xx}$$
$$U(0,t) = U_x(\ell,t) = 0$$
$$U(x,0) = 1$$
$$U_t(x,0) = \sin\frac{\pi x}{2\ell}$$

$$U(x,t) = \sum T_n(t)\, X_n(x)$$
$$T'' X = a^2 X'' T$$
$$\frac{T''}{a^2 T} = \frac{X''}{X} = -\lambda^2$$
$$X_n + \lambda^2 X_n = 0$$
$$X_n = A_n \sin\lambda_n x + B_n \cos\lambda_n x \xrightarrow{U(0,t)} B_n = 0$$
$$X_n' = \lambda_n A_n \cos\lambda_n x - B_n \lambda_n \sin\lambda_n x \xrightarrow{U(l,t)} A_n\lambda_n \cos\lambda_n \ell = 0 \Downarrow \cos\lambda_n \ell = 0$$
$$\begin{cases} \lambda_n = \frac{\pi}{2\ell}(2n-1) \\ X_n = A_n \sin\lambda_n x \end{cases} \quad n \in \mathbb{Z}$$

$$T'' + a^2\lambda^2 T = 0$$
$$T_n = \tilde{A}_n \cos(a,\lambda,t) + \tilde{B}_n \sin(a,\lambda_n,t)$$
$$T_n' = B_n \lambda_n \cos(a,\lambda,t) - A_n \lambda_n \sin(a,\lambda_n,t)$$
$$T_n(0) = 1 \Rightarrow \tilde{A}_n = 1$$
$$T_n'(0) = 1 \Rightarrow \tilde{B}_n = 1$$

$$U(x,t) = \sum_{n=1}^{\infty} A_n T_n X_n = \sum_n A_n \sin\lambda_n x (\sin(a,\lambda,t)) =$$
$$= \sum_n A_n \sin(\lambda,n,x) \sin(a,\lambda_n,t)$$
$$U_t(x,t) = A_n a \lambda_n \sin\lambda_n x \cos(\ell) = \sin\frac{\pi x}{2\ell}$$
$$\lambda_n = \frac{\pi}{2\ell}(2n-1)$$
$$A_n = \frac{2\ell}{\pi}$$
$$U(x,t) = \frac{2\ell}{\pi a}\sin\left(\frac{\pi x}{2\ell}\right)\sin\left(\frac{\pi a}{2\ell}t\right)$$

$$u(x,t) = \sin\frac{\pi x}{2\ell}\left[\frac{4}{\pi}\cos\left(\frac{\pi a t}{2\ell}\right) + \frac{2\ell}{\pi a}\sin\left(\frac{\pi a t}{2\ell}\right)\right] + \cancel{\sum} + \frac{4}{\pi}\sum_{n=1}^{\infty}(\dots)$$